P P SAVANI UNIVERSITY

First Semester of B. Tech. Examination

January 2023

SESH1070 Fundamentals of Mathematics

04.01.2023, Wednesday **Time: 10:00 a.m. To 12:30 p.m.** **Maximum Marks: 60**

Instructions:

1. The question paper comprises of two sections.
2. Section I and II must be attempted in separate answer sheets.
3. Make suitable assumptions and draw neat figures wherever required.
4. Use of scientific calculator is allowed.

SECTION – I

				CO	BTL
Q - 1	**Choose the correct answer:**		**[06]**		
(i)	Which of the following is not the condition of roll's theorem? a) The function must be continuous on given interval $[a, b]$. b) The function must be differentiable on given open interval (a, b). c) $f(a) = f(b)$ d) All of these			1	1/2
(ii)	$\lim_{x\to\infty} x^2 \sin\frac{1}{x} =$ ____ . a) 1 b) 0 c) ∞ d) None of these			1	2
(iii)	If $y = \cos(ax + b)$, then $y_{n=}$____. a) $y_n = a^n \cos\left(ax + b + \frac{n\pi}{2}\right)$ b) $y_n = b^n \sin\left(ax + b + \frac{n\pi}{2}\right)$ c) $y_n = a^n \sin\left(ax + b + \frac{n\pi}{2}\right)$ d) $y_n = \cos\left(ax + b + \frac{n\pi}{2}\right)$			1	1/2
(iv)	The limit of the sequence $S_n = \frac{3+2n}{n}$. a) 0 b) 1 c) 2 d) None of these			4	2
(v)	If $\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \infty$ then the series $\sum_{n=1}^{\infty} u_n$ is______. a) Convergent b) Divergent c) Oscillatory d) None of these			4	2
(vi)	Which of the following statement is true for p – test ? a) It converges if $p > 1$ and diverges to if $p \leq 1$. b) It converges if $p \leq 1$ and diverges to if $p > 1$. c) It converges only when $p = 1$. d) It is simply a divergent series.			4	5
Q - 2	**Answer the following questions: (Any four)**				
(i)	a) $\lim_{x\to 2} \frac{x^2-3x+2}{x^2-4}$	b) $\lim_{x\to\infty} \frac{3x^2-x-2}{5x^2+4x+1}$	**[06]**	1	5

(ii)	Find n^{th} derivative of $y = x\sin x$.	[06]	1	4/5
(iii)	Verify Lagrange's mean value theorem for $f(x) = x^2 + 2x + 3, x \in [4, 6]$.	[06]	1	4/5
	OR			
	Verify Cauchy's mean value theorem for $2x^3$ and $x^6, x \in [a, b], a > 0$.			
(iv)	Find the local extremum value of $f(x) = x^3 - 9x^2 + 15x + 11$.	[06]	1	5/6
(v)	Prove that $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \cdots$ converges and find its sum.	[06]	4	4/5
(vi)	Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2n-1}{n(n+1)(n+2)}$.	[06]	4	4/5
(vii)	Test the convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2(n+1)}$.	[06]	4	4/5

SECTION – II

Q - 1 Choose the correct answer: **[06]**

(i) If we put point $a = 0$ in Taylor series, then_________ 2 4

a) $f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!} f''(a) + \cdots$

b) $f(x) = f(a) + (x-a)f''(a) + \frac{(x-a)^2}{2!} f''(a) + \cdots$

c) $f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!} f'''(a) + \cdots$

d) None of these

(ii) The value of $\lim_{x\to 0} \frac{\sin 2x}{x} =$ ____. 2 5

a) 0 b) 1 c) -1 d) 2

(iii) The Maclaurin's series of $f(x)$ is

a) $f(x) + \frac{x}{1!} f'(x) + \frac{x^2}{2!} f''(x) + \cdots + \frac{x^n}{n!} f^n(x) + \cdots$

b) $f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \cdots + \frac{x^n}{n!} f^n(0) + \cdots$

c) $1 + \frac{x}{1!} f'(1) + \frac{x^2}{2!} f''(1) + \cdots + \frac{x^n}{n!} f^n(1) + \cdots$

d) $1 + \frac{x}{1!} f'(x) + \frac{x^2}{2!} f''(x) + \cdots + \frac{x^n}{n!} f^n(x) + \cdots$

(iv) Find the eigen value of $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$. 2 3/5

a) $-7, -3$ b) $7, -3$ c) $7, 3$ d) $-7, 3$

(v) The eigen values of A are $3, 3, -3$, so the eigen values of A^{-1} is ____. 3 3/5

a) $3, 3, -3$ b) $\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}$ c) $3, -3, -3$ d) $-3, -3, -3$

(vi) f $A = \begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix}$ then the eigen values of $A^3 =$ ____. 3 3

a) 1, 4 b) $-1, 4$ c) 1, 64 d) $1, -4$

Q - 2 Answer the following questions: (Any four)

(i)	Evaluate $\lim_{x\to 2}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$.	[06]	2	5
(ii)	Expand $\sin x$ in powers of $\left(x-\frac{\pi}{2}\right)$ by Taylor's series. Hence find the value of $\sin 91°$.	[06]	2	5/6
(iii)	Use Maclaurin's series to determine the expansion of $(3+2t)^4$.	[06]	2	4/5
(iv)	Find the inverse of the matrix $A=\begin{bmatrix}1&0&1\\-1&1&1\\0&1&0\end{bmatrix}$ by Gauss-Jordan method.	[06]	3	5
(v)	Solve the following system of linear equation by Gauss elimination method: $x+2y-3z=-2;$ $3x-y-2z=1;$ $2x+3y-5z=-3.$	[06]	3	5
(vi)	Find eigen value and eigen vector of $\begin{bmatrix}4&6&6\\1&3&2\\-1&-5&-2\end{bmatrix}$.	[06]	3	5
(vii)	Verify Cayley-Hamilton theorem for $A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$ and hence find A^{-1}.	[06]	3	3/5

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CO : Course Outcome Number BTL : Blooms Taxonomy Level

Level of Bloom's Revised Taxonomy in Assessment

1: Remember	2: Understand	3: Apply
4: Analyze	5: Evaluate	6: Create